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SURVEILLANCE PROGRAMS FOR DETERIORATING LOTS IN STORAGE

By

FREDERICK S. HILLIER

TECHNICAL REPORT NO. 56

JANUARY 2, 1962

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1. Introduction.

The military and many civilian organizations require that a sufficient supply of materials be in storage at any one time to meet certain emergency needs that may arise. The fulfillment of this requirement is primarily a procurement problem. However, the procurement of an adequate supply of materials alone does not necessarily guarantee an adequate supply of usable materials when the need for them arises. Their deterioration in storage may render them unusable. Despite all precautions, at least a small rate of deterioration is inevitable for all items in storage. It is therefore essential that a systematic program for the surveillance of such stored items be maintained in order to insure that the supply of usable items is always adequate.

The development of such surveillance programs has received very little attention in the literature. Derman and Solomon [1] studied the quality of lots in storage over time when these lots are periodically subjected to standard acceptance sampling plans. Ireson [2] discussed administrative aspects of surveillance programs. Savage [3] briefly examined when a piece of equipment reserved for emergency use should be withdrawn from storage for repair; this paper considered in general terms the case where the relevant costs are those for regular repair and for repair when the emergency occurs and both these costs are known deterministic functions of the time elapsed since the last repair. However, no comprehensive investigation has yet been reported regarding the appropriate time for corrective action to compensate for the deteriorating quality of a lot in storage, given the cost of appropriate corrective action and the cost of an imperfect lot when the emergency occurs as functions of the number of defective items in the lot. This paper presents the results of such an investigation.

The assumption made about the lot in storage is that the storage life of each item, i.e., the time in storage until becoming defective, has a common known exponential distribution. Of course, this condition is not always met. However, the exponential distribution of life has been demonstrated and justified on empirical and theoretical grounds for numerous types of items. Furthermore, the particular exponential distribution can often be estimated by experimentation or from prior experience with the same product. Thus, this formulation will often become applicable at some time after the initiation of a continuing surveillance program for a given product.

It is also assumed that the time until the emergency necessitating the use of the lot has a known exponential distribution. This distribution should ordinarily be the appropriate one since it implies the usual situation that the emergency is a random event in time. Examples of such emergencies that might be of interest are the beginning of a war, a natural disaster, the breakdown or failure of certain equipment, the need to enter a fallout shelter, a rush order, the rejection of regular material, and the cancellation of needed deliveries by a supplier.

With the information available regarding the quality of the lot over time, it is no longer necessary to sample the lot periodically to estimate its quality in order to determine if any corrective action is advisable. It is now practical to instead select the future time when the deteriorating quality of the lot will justify corrective action, if the emergency has not occurred yet. This corrective action might involve inspecting the entire lot and replacing or repairing those items found to be deficient. However, if inspection is sufficiently costly, or if it involves destructive testing, or if the future quality of the non-defectives in the lot is mistrusted-despite the assumption of an exponential storage life- the proper action would be to immediately replace or repair the entire lot. The formulation presented here permits both of these alternatives.

In the absence of any surveillance sampling, there remain only two types of costs that are not independent of the surveillance program selected. One type is the cost of taking corrective action periodically until the emergency occurs. Its expected value tends to decrease as the time interval between successive corrective actions is increased. The second type is the cost or the imputed cost incurred because of the defective items in the lot when the emergency occurs. The expected value of this type of cost increases as the time between corrective actions is increased. Therefore, this time needs to be carefully selected so as to obtain the proper balance between these two types of costs. To do so requires that the cost of corrective action and the cost of using an imperfect lot, each as a function of the number of the defectives in the lot, be known.

The problem considered here is the determination of the time between corrective actions which minimizes the total expected cost from the beginning of the program until the emergency is met. Section 2 presents the formulation of the problem and the operating characteristics of a given surveillance program. The critical equation is derived for general cost functions in Section 3. It is then developed and simplified for specific cost functions of interest in the following two sections. These results are illustrated with an example in Section 6. The discussion of the results in Section 7 includes a suggestion of how they can also be applied in the initial absence of information regarding the distribution of storage life. The final section summarizes the conclusions.

2. Formulation of the Problem.

A lot composed of N items is being held in storage. It will be used only if certain emergency needs arise, as in the stockpiling of vital material. The probability density function of the time until such an emergency arises, $g(t)$, is given by

$$g(t) = \begin{cases} 0 \quad , & \text{if } t < 0 \\ \theta e^{-\theta t} \quad , & \text{if } t \geq 0 , \end{cases}$$

where $\theta > 0$ and is known. The quality of the lot can be expected to deteriorate with age. At periodic intervals, corrective action is taken. This corrective action consists either of replacing the entire lot or of inspecting each item in the lot and either replacing or repairing those items found to be defective. The known cost of taking corrective action when there are n defective items in the lot is $R(n)$. For each item in the lot, the cumulative density function of the time elapsed since the last corrective action until the item becomes defective, $F(t)$, is given by

$$F(t) = 1 - e^{-\mu(t+t_o)} ,$$

where $1 - e^{-\mu t_o}$ is the known initial probability of being defective and μ has a known positive value. Thus, at time t, the defective vs. non-defective status of the respective items can be considered as independent Bernoulli trials. Hence, the probability that the lot contains n defective items at time t, the time elapsed since the last corrective action, is given by

$$P_n(t) = \begin{cases} \binom{N}{n} \left(1 - e^{-\mu(t+t_o)}\right)^n \left(e^{-\mu(t+t_o)}\right)^{N-n} , & \text{if } n = 0, 1, 2, \ldots ,N \\ 0 , & \text{otherwise.} \end{cases}$$

The known cost or imputed cost resulting from the presence of n defective items in the lot at the time the emergency arises is $D(n)$. The objective is to determine the value of t, the elapsed time between successive corrective actions, which minimizes the total expected cost from the beginning of the program until the emergency is met.

3. Analysis

Let $T = (t_1, t_2, \ldots)$ be the sequence of positive numbers such that corrective action is taken at the times $t_1, t_1 + t_2, \ldots$, unless the emergency intervenes. Let the random variable E be the time at which the emergency occurs. Thus, corrective action is taken at the times $t_1, t_1 + t_2, \ldots, \sum_{i=1}^{m} t_i$, where $\sum_{i=1}^{m} t_i < E \leq \sum_{i=1}^{m+1} t_i$.

The total cost from the beginning of the program[1/] until the emergency is met is

$$C(T) = \sum_{i=1}^{m} R(n_i) + D(n_E) \quad ,$$

where $n_1, n_2, \ldots, n_m$ are the number of defectives in the lot at times $t_1, t_1 + t_2, \ldots, \sum_{i=1}^{m} t_i$, and n_E is the number of defectives in the lot at time E. Let $C_o(T)$ be the cost from the beginning of the program until and including the time t_1. Let $C_1(T)$ be the cost of the program after the time t_1. Therefore,

$$C(T) = C_o(T) + C_1(T) \quad .$$

Since

$$C_1(T|m = 0) = 0 \ ,$$

$$C_1(T) = C_1(T|m > 0)\ P(m > 0) \quad .$$

[1/] It is assumed that the program is begun immediately after corrective action has been taken. Corrective action may be defined to include the initial placement of the lot in storage if the effect on the quality of the lot is indistinguishable.

Hence,

$$E\{C(T)\} = E\{C_o(T)\} + E\{C_1(T|m > 0)\} P(m > 0) \quad .$$

Let

$$S_1 = \{T | E\{C(T)\} = \inf_{X \epsilon S_2} E\{C(X)\},\ T \epsilon S_2\} \quad ,$$

where

$$S_2 = \{(t_1,\ t_2,\ \dots\ ) \ |\ t_i > 0,\ i = 1, 2, \dots\ \} \quad .$$

It is assumed that S_1 is not empty, i. e., that there exists a solution to the problem. The objective is to find a solution by determining an element of S_1.

Notice that

$$P(m > 0) = P(E > t_1) = e^{-\theta t_1} \quad .$$

Therefore, forming the expectation for $C_o(T)$, $E\{C(T)\}$ becomes

$$E\{C(T)\} = \int_o^{t_1} (\sum_{j=o}^{N} D(j)\ P_j(t))\theta e^{-\theta t}\ dt$$

$$+ \ e^{-\theta t_1} \sum_{j=o}^{N} R(j) P_j(t_1) + e^{-\theta t_1} E\{C_1(T|m > 0)\} \ .$$

Due to the nature of the exponential distribution, i. e.,

$$P(E > t_1 + t \mid E > t_1) = P(E > t) \quad ,$$

and also because the quality of the lot after t_1 is independent of its quality before t_1, it is clear that $C_1(T \mid m > 0)$, and therefore $E\{C_1(T \mid m > 0)\}$, is independent of t_1. Hence, it is evident that $E\{C(T)\}$ is a function of t_1 and $E\{C_1(T \mid m > 0)\}$, where $E\{C_1(T \mid m > 0)\}$ is a function of $t_2, t_3, \ldots$, and that if $(t_1, t_2, t_3, \ldots)$ minimizes $E\{C(T)\}$, i.e., if $(t_1, t_2, t_3, \ldots) \epsilon S_1$, then $(t_1, t_2, t_3, \ldots)$ minimizes $E\{C_1(T \mid m > 0)\}$. Therefore, if $(t_1, t_2, t_3, \ldots) \epsilon S_1$, then if $(t_1, t_2', t_3', \ldots)$ also minimizes $E\{C_1(T \mid m > 0)\}$, $(t_1, t_2', t_3', \ldots) \epsilon S_1$. It also follows from the nature of the exponential distribution and the fact that the quality of the lot depends only on the time elapsed since the last corrective action that if $(t_1, t_2, t_3, \ldots) \epsilon S_1$, then $(t_1, t_1, t_2, \ldots)$ minimizes $E\{C_1(T \mid m > 0)\}$. Therefore if $(t_1, t_2, t_3, \ldots) \epsilon S_1$, then $(t_1, t_1, t_2, \ldots) \epsilon S_1$. By repeated application of this argument, it follows that if $(t_1, t_2, t_3, \ldots) \epsilon S_1$, then $(t_1, t_1, t_1, \ldots) \epsilon S_1$. The important conclusion is that one solution to this problem is of the form that the time between corrective actions is always the same. This is the solution that will be sought. Thus denoting this constant positive time between consecutive corrective actions by t_1, the problem now consists of determining the optimum value of t_1.

It will prove very useful to notice that, for this newly defined problem, $E\{C(T)\} = E\{C_1(T \mid m > 0)\}$. Since T is now restricted to be of the form $(t_1, t_1, t_1, \ldots)$ where $t_1 > 0$, it will be more meaningful to now denote $E\{C(T)\}$ by $E\{C(t_1)\}$. The expression for the total expected cost can now be written as

$$E\{C(t_1)\} = \int_0^{t_1} \left(\sum_{n=0}^{N} D(n)\, P_n(t)\right)\theta e^{-\theta t} dt$$

$$+ e^{-\theta t_1}\left[\sum_{n=0}^{N} R(n)P_n(t_1) + E\{C(t_1)\}\right] .$$

Therefore,

$$\left(1-e^{-\theta t_1}\right) E\{C(t_1)\} = \int_0^{t_1} \left(\sum_{n=0}^{N} D(n)\, P_n(t)\right)\theta_e{}^{-\theta t} dt$$

$$+ e^{-\theta t_1} \sum_{n=0}^{N} R(n)P_n(t_1) \quad ,$$

so that

$$E\{C(t_1)\} = \frac{1}{1-e^{-\theta t_1}}\left[\int_0^{t_1} \left(\sum_{n=0}^{N} D(n)P_n(t)\right)\theta e^{-\theta t} dt + e^{-\theta t_1} \sum_{n=0}^{N} R(n)P_n(t_1)\right] .$$

For the particular $P_n(t)$ function for this problem, specified earlier, the conditions for Liebnitz's Rule are met and the derivative exists. Setting the derivative equal to zero,

$$\frac{d}{dt_1}\Big(E\{C(t_1)\}\Big) = \frac{\sum_{n=o}^{N} D(n)P_n(t_1)\theta e^{-\theta t_1}}{1-e^{-\theta t_1}} - \left[\frac{\theta e^{-\theta t_1}}{\left(1-e^{-\theta t_1}\right)^2}\right]\int_o^{t_1}\left(\sum_{n=o}^{N} D(n)P_n(t)\right)\theta e^{\theta t}dt$$

$$+ \frac{e^{\theta t_1}\frac{d}{dt_1}\left(\sum_{n=o}^{N} R(n)P_n(t_1)\right) - \theta e^{-\theta t_1}\sum_{n=o}^{N} R(n)P_n(t_1)}{1-e^{-\theta t_1}}$$

$$- \left[\frac{\theta e^{-\theta t_1}}{\left(1-e^{-\theta t_1}\right)^2}\right] e^{-\theta t_1}\sum_{n=o}^{N} R(n)P_n(t_1)$$

$$= 0 \quad .$$

Cancelling common terms, the critical equation becomes

$$\left[1-e^{-\theta t_1}\right]\left[\sum_{n=o}^{N} D(n)P_n(t_1) + \frac{1}{\theta}\frac{d}{dt_1}\left(\sum_{n=o}^{N} R(n)P_n(t_1)\right) - \sum_{n=o}^{N} R(n)P_n(t_1)\right]$$

$$= \int_o^{t_1}\sum_{n=o}^{N} D(n)P_n(t)\,\theta e^{-\theta t}\,dt + e^{-\theta t_1}\sum_{n=o}^{N} R(n)P_n(t_1) \quad .$$

Simplifying slightly,

$$\left[1 - e^{-\theta t_1}\right]\left[\sum_{n=o}^{N} D(n)P_n(t_1) + \frac{1}{\theta}\frac{d}{dt_1}\sum_{n=o}^{N} R(n)P_n(t_1)\right]$$

$$= \int_o^{t_1}\left(\sum_{n=o}^{N} D(n)P_n(t)\right)\theta e^{-\theta t}\,dt + \sum_{n=o}^{N} R(n)P_n(t_1) \quad .$$

It is apparent that this critical equation would be exceedingly difficult to solve for t_1 for most $D(n)$ and $R(n)$ functions. But if one were unable to solve this equation, the analysis would have little practical value. Fortunately, greatly simplified equations quite amenable to solution can be obtained for certain $D(n)$ and $R(n)$ functions of great practical interest. This is especially true where $D(n)$ and $R(n)$ are linear functions of n. This case is studied and a comprehensive set of solutions graphed in the next section. The following section then studies the case where $R(n)$ is a linear function and $D(n)$ is a certain quadratic function.

4. Solution for Linear $D(n)$ and $R(n)$ Functions

Assume that $D(n) = C_D \cdot n$. In other words, the cost of having defectives in the lot when the emergency occurs is proportional to n, the number of those defectives. This would seem to be the appropriate $D(n)$ function for many situations. It should also serve as a suitable approximation for many more situations.

Assume that $R(n) = K + C_R \cdot n$, where $K > 0$. In other words, the cost of taking corrective action-whether it be the replacement of the entire lot, or more likely the screening of the lot and the repair or replacement of defective items - consists of a fixed cost plus a certain cost for each defective item in the lot. This would appear to be the appropriate $R(n)$ function, or at least a reasonable approximation, for practically every situation. The fixed cost could be the red tape cost of ordering or producing new items plus the cost of inspecting the lot. The cost for each defective item could be the cost associated only with replacing or repairing that item. If the lot is automatically replaced, the entire cost is K.

When performing the summation operations for this case, it is crucial to notice that, for fixed t, the $P_n(t)$ probabilities describe a binomial distribution with parameter $(1-e^{-\mu(t+t_o)})$, and that the mean of this distribution is $N(1-e^{-\mu(t+t_o)})$. The $\sum_{n=o}^{N} D(n)P_n(t)$ and $\sum_{n=o}^{N} R(n)P_n(t)$ summations are analogous to finding this mean.

Thus,

$$\sum_{n=o}^{N} D(n)P_n(t) = \sum_{n=o}^{N} C_D \cdot n\binom{N}{n} \left(1-e^{-\mu(t+t_o)}\right)^n \left(e^{-\mu(t+t_o)}\right)^{N-n}$$

$$= C_D \cdot N\left(1-e^{-\mu(t+t_o)}\right) ,$$

and

$$\sum_{n=o}^{N} R(n)P_n(t) = \sum_{n=o}^{N} (K+C_R \cdot n) \binom{N}{n} \left(1-e^{-\mu(t+t_o)}\right)^n \left(e^{-\mu(t+t_o)}\right)^{N-n}$$

$$= K + C_R \cdot N\left(1-e^{\mu(t+t_o)}\right) .$$

Therefore, after the indicated differentiation and integration operations are performed, the critical equation becomes

$$\left[1-^{-\theta t_1}\right] \left[C_D \cdot N \left(1-e^{-\mu(t_1+t_o)}\right) + \frac{1}{\theta} C_R \cdot N\mu e^{-\mu(_1+t_o)}\right]$$

$$= C_D \cdot N\left[1-e^{\theta t_1} - \frac{\theta}{\mu+\theta} e^{-\mu t_o}\left(1-e^{-t_1(\mu+\theta)}\right)\right] + \left[K+C_R \cdot N\left(1-e^{-\mu(t+t_o)}\right)\right].$$

Combining like terms and simplifying,

$$\left(1-e^{-\theta t_1}\right) e^{-\mu t_1} \left(-C_D \cdot N + \frac{C_R \cdot N\mu}{\theta}\right) + C_R \cdot Ne^{-\mu t_1} - C_D \cdot N \frac{\theta}{\mu+\theta} e^{-t_1(\mu+\theta)}$$

$$= e^{\mu t_o} \left[K+C_R \cdot N - C_D \cdot Ne^{-\mu t_o} \left(\frac{\theta}{\mu+\theta}\right)\right] .$$

Simplifying further and dividing through by N,

$$e^{-\mu t_1} \left[\left(-C_D + \frac{C_R\mu}{\theta} + C_R\right) - \left(-C_D + \frac{C_R\mu}{\theta} + C_D \frac{\theta}{\mu+\theta}\right) e^{-\theta t_1}\right]$$

$$= e^{\mu t_o} \left[\frac{K}{N} + C_R - C_D e^{-\mu t_o} \left(\frac{\theta}{\mu+\theta}\right)\right] .$$

Finally,

$$e^{-\mu t_1} \left[\left(-C_D + \frac{\mu+\theta}{\theta} C_R\right) + \left(\frac{\mu}{\mu+\theta} C_D - \frac{\mu}{\theta} C_R\right) e^{-\theta t_1}\right]$$

$$= e^{\mu t_o} \left[\frac{K}{N} + C_R - C_D e^{-\mu t_o} \left(\frac{\theta}{\mu+\theta}\right)\right] .$$

In order to simplify notation, let

$$D = e^{\mu t_o}\left[\frac{K}{N} + C_R - C_D e^{-\mu t_o}\left(\frac{\theta}{\mu+\theta}\right)\right] ;$$

$$A = \frac{-C_D + \frac{\mu+\theta}{\theta} C_R}{D} , \qquad \text{if} \quad D \neq 0 \ ;$$

$$B = \frac{\frac{\mu}{\mu+\theta} C_D - \frac{\mu}{\theta} C_R}{D} , \qquad \text{if} \quad D \neq 0 \ .$$

Thus, assuming $D \neq 0$, the critical equation can now be written as

$$e^{-\mu t_1}\left[A + Be^{-\theta t_1}\right] = 1.$$

Notice that

$$B = -\left(\frac{\mu}{\mu+\theta}\right) A \ .$$

Therefore, the most convenient form of the critical equation is

$$Ae^{-\mu t_1}\left[1 - \left(\frac{\mu}{\mu+\theta}\right)e^{-\theta t_1}\right] = 1 \ .$$

Notice that it has a positive solution only if $A > 1$. While an explicit solution for t_1 is not available, this critical equation can be easily solved for t_1 by numerical methods. This is especially true since

$$\frac{d}{dt_1}\left(Ae^{\mu t_1}\left[1-\left(\frac{\mu}{\mu+\theta}\right)e^{-\theta t_1}\right]\right)$$

$$= -A\mu e^{-\mu t_1} + A\left(\mu+\theta\right)\left(\frac{\mu}{\mu+\theta}\right)e^{-t_1(\mu+\theta)}$$

$$= -A\mu e^{-\mu t_1}\left(1 - e^{-\theta t_1}\right)$$

$$< 0 \quad , \qquad\qquad \text{for } t_1 > 0, \ A > 0 \ .$$

Thus, if $A > \frac{\mu+\theta}{\theta}$, the critical equation has a unique positive solution; otherwise, it has no positive root. Furthermore, because the left side of the equation strictly decreases as t_1 increases, t_1 can be estimated as closely as desired by determining lower and upper bounds to t_1 by trial and error.

Figures 4.1, 4.2, and 4.3 present a comprehensive set of solutions of the critical equation. For the particular values of $(A - \frac{\mu}{\theta})$ and $(\frac{\mu}{\mu+\theta})$ of interest, the value of μt_1 and therefore t_1 can be closely estimated by interpolating between the given level curves.

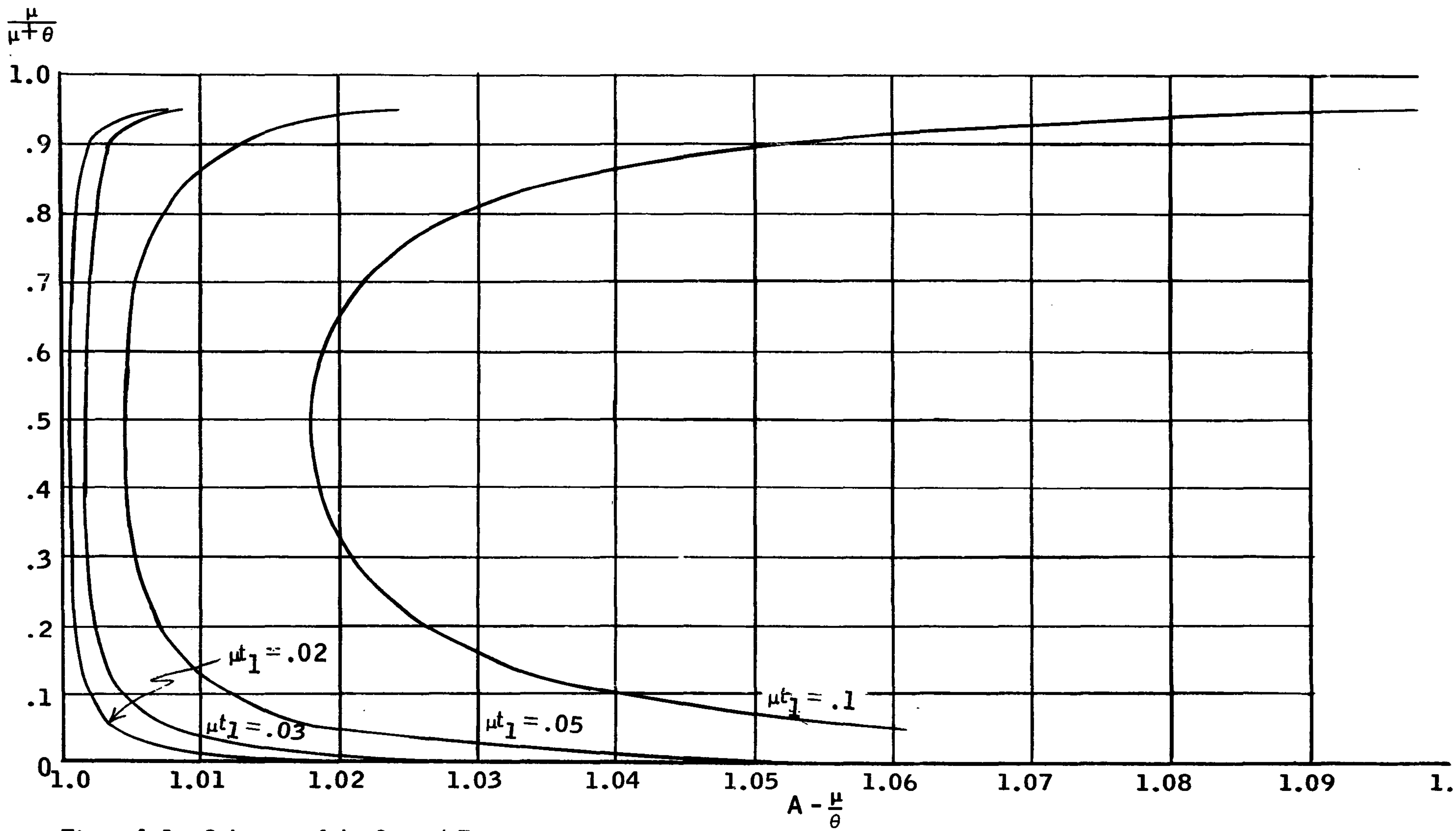

Figure 4.1. Solutions of the Critical Equation.

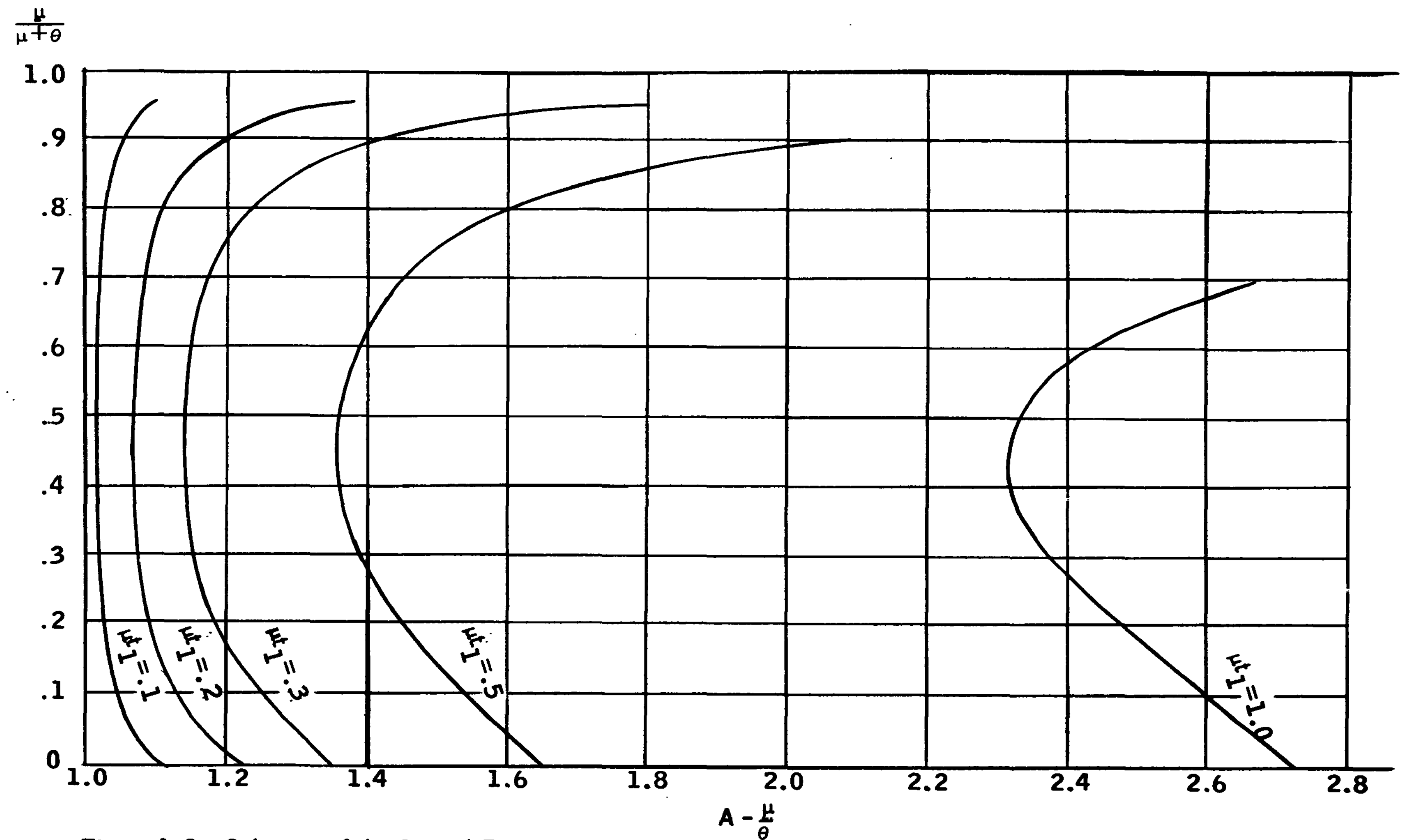

Figure 4.2. Solutions of the Critical Equation.

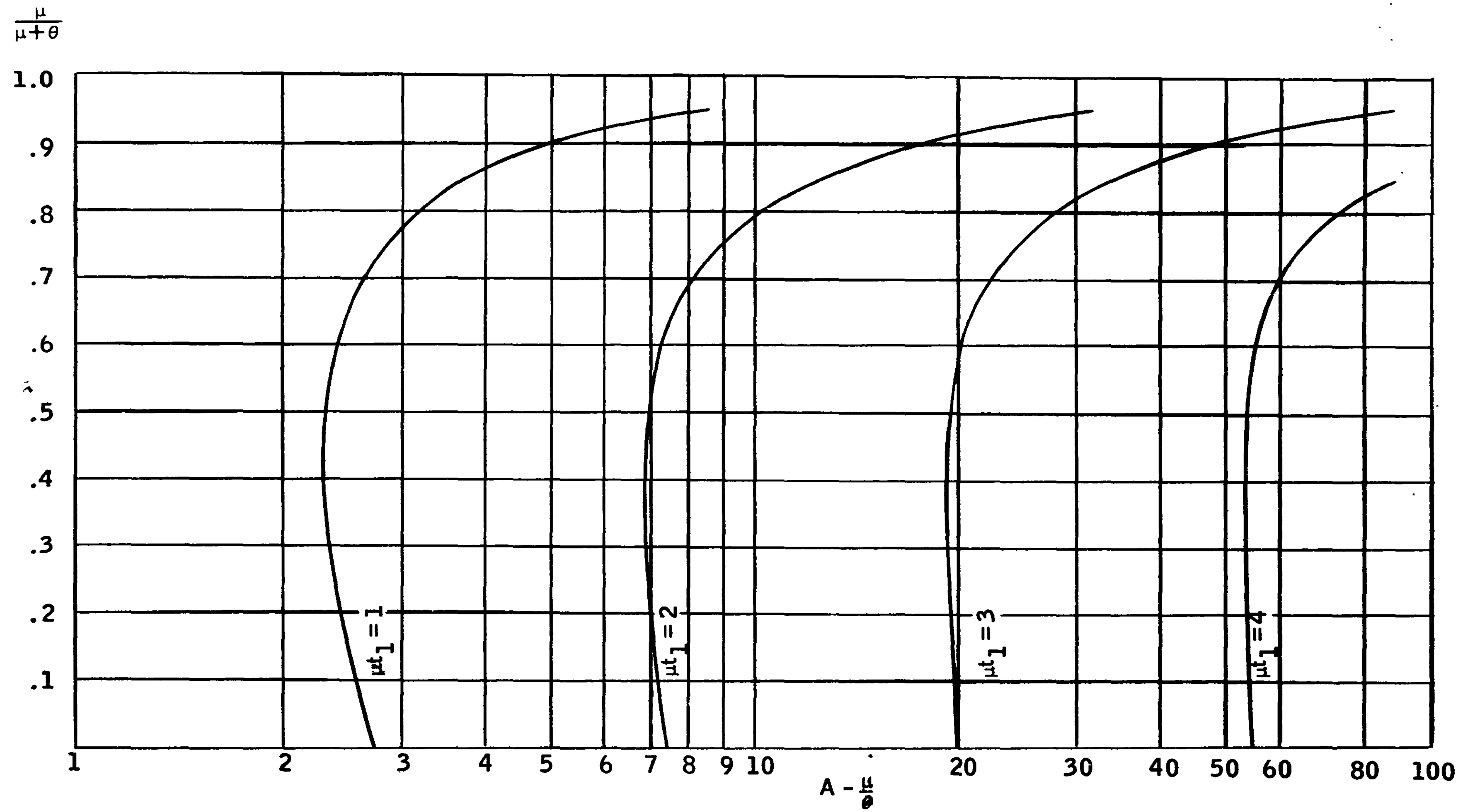

Figure 4.3. Solutions of the Critical Equation.

It is of course true that being a positive root of the critical equation is a necessary, but not sufficient, condition for minimizing $E\{C(t_1)\}$. The sufficient condition, that

$$\frac{d^2}{dt_1^2}\left(E\{C(t_1)\}\right) > 0 \quad ,$$

will now be investigated. Assume that $A > \frac{\mu+\theta}{\theta}$, $D \neq 0$.

Tracing through the derivation of the critical equation, it is clear that

$$\frac{d}{dt_1}\left(E\{C(t_1)\}\right) = D\,\frac{\theta e^{-\theta t_1}}{\left(1-e^{\theta t_1}\right)^2}\left(Ae^{-\mu t_1}\left[1-\left(\frac{\mu}{\mu+\theta}\right)e^{-\theta t_1}\right] - 1\right) Ne^{-\mu t_o} \quad .$$

Therefore,

$$\frac{d^2}{dt_1^2}\left(E\{C(t_1)\}\right) = D\,\frac{\theta e^{-\theta t_1}}{\left(1-e^{-\theta t_1}\right)^2}\left(-A\mu e^{-\mu t_1}\left[1-e^{-\theta t_1}\right]\right) Ne^{-\mu t_o}$$

$$+ D\left(\frac{-(1-e^{-\theta t_1})^2\,\theta^2\,e^{-\theta t_1}-\theta e^{-\theta t_1}\cdot 2(1-e^{-\theta t_1})\theta e^{-\theta t_1}}{\left(1-e^{-\theta t_1}\right)^4}\right)\left(Ae^{-\mu t_1}\left[1-\frac{\mu}{\mu+\theta}e^{-\theta t_1}\right]-1\right)Ne^{-\mu t_o}.$$

Assume that t_1 is a positive root of the critical equation. Hence, the second term vanishes. Additionally,

$$\frac{\theta e^{-\theta t_1}}{\left(1-e^{-\theta t_1}\right)^2} \left(-A\mu e^{-\mu t_1} \left[1-e^{-\theta t_1}\right]\right) Ne^{-\mu t_o} < 0 \quad .$$

Therefore,

$$\frac{d^2}{dt_1^2} \left(E\{C(t_1)\}\right) > 0$$

if and only if $D < 0$. In other words, if

$$\frac{K}{N} + C_R < C_D e^{-\mu t_o} \left(\frac{\theta}{\mu+\theta}\right) \quad ,$$

then the t_1^* which is the positive root of the critical equation minimizes $E\{C(t_1)\}$. This is further verified by the fact that, for this case,

$$\frac{d}{dt_1}\left(E\{C(t_1)\}\right) < 0 \qquad\qquad \text{for} \quad 0 < t_1 < t_1^* \ .$$

It remains to investigate the other cases. If $D > 0$ it becomes quickly clear that this implies $A \leq \frac{\mu+\theta}{\theta}$, which in turn implies

$$\frac{d}{dt} \left(E\{C(t_1)\}\right) < 0 \qquad\qquad \text{for} \quad t_1 > 0 \quad ,$$

which indicates that corrective action should never be taken, i.e., $t_1 = \infty$. This is intuitively reasonable since $D > 0$ implies that the cost of corrective action is high relative to the cost of having defectives in the lot when the emergency occurs.

It is easily seen that the case that $A \leq \frac{\mu+\theta}{\theta}$ can not occur if $D < 0$. Therefore, the only remaining case is $D = 0$. For this case, the critical equation becomes

$$\left(-C_D + \frac{\mu+\theta}{\theta} C_R\right) e^{-\mu t_1} \left[1 - \left(\frac{\mu}{\mu+\theta}\right) e^{-\theta t_1}\right] = 0 \ .$$

But this equation has no positive root since

$$-C_D + \frac{\mu+\theta}{\theta} C_R < \frac{\mu+\theta}{\theta} D = 0 \ ,$$

since $K > 0$. It is similarly verified that, for this case,

$$\frac{d}{dt_1}\left(E\{C(t_1)\}\right) = \left(-C_D + \frac{\mu+\theta}{\theta} C_R\right) Ne^{-\mu t_o} \frac{\theta e^{-\theta t_1}}{\left(1-e^{-\theta t_1}\right)^2} e^{-\mu t_1}\left[1-\left(\frac{\mu}{\mu+\theta}\right) e^{-\theta t_1}\right]$$

$$< 0 \qquad\qquad \text{for } t_1 > 0 \ .$$

Therefore, corrective action should never be taken.

The general conclusion is that the t_1 which minimizes $E\{C(t_1)\}$ is the positive root of the critical equation if a positive root exists; if the positive root exists, it will be unique; if it does not exist, t_1 should be infinite, i.e., never take corrective action.

5. Solution for Certain Quadratic $D(n)$ and Linear $R(n)$ Functions

Assume that $D(n) = C_D \cdot n^2$. This should serve as a suitable approximation for the type of situation in which the availability of non-defective items in the lot can partially compensate for the defective items, so that the imputed cost of having only a few defectives in the lot is relatively low but this cost increases very rapidly as the number of defectives increases. Assume, as before, that $R(n) = K + C_R \cdot n$, where $K > 0$.

Proceeding as before, it needs to be recalled that the second moment of the binomial distribution described by the $P_n(t)$ probabilities is

$$(N^2 - N)\left(1-e^{-\mu(t+t_o)}\right)^2 + N\left(1-e^{-\mu(t+t_o)}\right) .$$

The $\sum_{n=o}^{N} D(n)P_n(t)$ summation is analogous to finding this second moment. Thus,

$$\sum_{n=o}^{N} D(n)P_n(t) = \sum_{n=o}^{N} C_D \cdot n^2 \binom{N}{n} \left(1-e^{-\mu(t+t_o)}\right)^n \left(e^{-\mu(t+t_o)}\right)^{N-n}$$

$$= C_D \left[(N^2 - N)\left(1-e^{-\mu(t+t_o)}\right)^2 + N\left(1-e^{-\mu(t+t_o)}\right)\right]$$

$$= NC_D \left[(N-1)e^{-2\mu(t+t_o)} + (1-2N)e^{-\mu(t+t_o)} + N\right] .$$

Therefore, proceeding in a straight-forward fashion and simplifying,

$$\int_0^{t_1} \sum_{n=0}^{N} D(n)P_n(t)\theta e^{-\theta t}\, dt$$

$$= NC_D\left[\frac{(N-1)\theta e^{-2\mu t_o}}{2\mu+\theta}\left(1-e^{-t_1(2\mu+\theta)}\right) + \frac{(1-2N)\theta e^{-\mu t_o}}{\mu+\theta}\left(1-e^{-t_1(\mu+\theta)}\right) + N\left(1-e^{-t_1\theta}\right)\right]$$

Hence, after considerable simplification,

$$\frac{d}{dt_1}\left(E\{C(t_1)\}\right) = \frac{\theta e^{-\theta t_1} Ne^{-\mu t_o}}{\left(1-e^{-\theta t_1}\right)^2} Q ,$$

where

$$Q = C_D(N-1)e^{-\mu(t_o+2t_1)}\left(1 - \frac{2\mu}{2\mu+\theta} e^{-\theta t_1}\right)$$

$$+ C_D(1-2N)e^{-\mu t_1}\left(1- \frac{\mu}{\mu+\theta} e^{-\theta t_1}\right)$$

$$+ C_R \frac{\mu}{\theta} e^{-\mu t_1}\left(1-e^{\theta t_1}\right) + C_R e^{-\mu t_1}$$

$$- \left[\frac{K}{Ne^{-\mu t_o}} + \frac{C_R}{e^{-\mu t_o}} + C_D(N-1)\, e^{-\mu t_o}\left(\frac{\theta}{2\mu+\theta}\right) + C_D(1-2N)\left(\frac{\theta}{\mu+\theta}\right)\right].$$

Thus, the critical equation is

$$Q = 0 \quad .$$

Assume that t_1^* is a positive root of the critical equation. Thus, if

$$\frac{d^2}{dt_1^2}\left(E\{C(t_1)\}\right)\Bigg|_{t_1=t_1^*} > 0 ,$$

then $E\{C(t_1^*)\}$ is at least a local minimum.

$$\frac{d^2}{dt_1^2}\left(E\{C(t_1)\}\right) = \frac{\theta e^{-\theta t_1} Ne^{-\mu t_o}}{\left(1-e^{-\theta t_1}\right)^2}\,\frac{dQ}{dt_1}$$
$$- Ne^{-\mu t_o}\left[\frac{\left(1-e^{-\theta t_1}\right)^2 \theta^2 e^{-\theta t_1} + 2\theta^2 e^{-2\theta t_1}\left(1-e^{-\theta t_1}\right)}{\left(1-e^{-\theta t_1}\right)^4}\right] Q .$$

Hence,

$$\frac{d^2}{dt_1^2}\left(E\{C(t_1)\}\right)\Bigg|_{t_1=t_1^*} = \frac{\theta e^{-\theta t_1^*} Ne^{-\mu t_o}}{\left(1-e^{-\theta t_1^*}\right)^2}\,\frac{dQ}{dt_1}\Bigg|_{t_1=t_1^*} .$$

Thus, if

$$\frac{dQ}{dt_1}\Bigg|_{t_1=t_1^*} > 0 ,$$

then $E\{C(t_1^*)\}$ is a local minimum. After considerable simplification, $\frac{dQ}{dt_1}$ can be written as

$$\frac{dQ}{dt_1} = \mu e^{-\mu t_1}\left(1-e^{-\theta t_1}\right)\left[2C_D(N-1)\left(1-e^{-\mu(t_o+t_1)}\right) + C_D - C_R\left(\frac{\mu+\theta}{\theta}\right)\right].$$

Let t_1' be the value of t_1 such that

$$2C_D(N-1)\left(1-e^{-\mu(t_o+t_1')}\right) + C_D = C_R\left(\frac{\mu+\theta}{\theta}\right) .$$

Then, clearly, if $t_1 > 0$, then

$$\frac{dQ}{dt_1} < 0 , \qquad \text{if} \quad t_1 < t_1' ;$$

$$\frac{dQ}{dt_1} > 0 , \qquad \text{if} \quad t_1 > t_1' .$$

It now needs to be pointed out that Q is continuous and that $Q < 0$ at $t_1 = 0$. Therefore, there exists $\epsilon > 0$ such that

$$\frac{d}{dt_1}\left(E\{C(t_1)\}\right) < 0 \qquad \text{if} \quad 0 < t_1 < \epsilon .$$

Hence, since $\frac{dQ}{dt_1} < 0$ if $t_1 > 0$ and $t_1 < t_1'$,

$$\frac{d}{dt_1}\left(E\{C(t_1)\}\right) < 0 \qquad \text{if} \quad t_1 > 0 \quad \text{and} \quad t_1 \leq t_1' .$$

Therefore, if $t_1 > 0$ and

$$\frac{d}{dt_1}(E\{C(t_1)\}) = 0 \quad ,$$

then $t_1 > t_1'$, and hence, $\frac{dQ}{dt_1} > 0$. The implication is that if t_1^* is a positive root of the critical equation, then $E\{C(t_1^*)\}$ must be a local minimum. Furthermore, since $\frac{dQ}{dt_1} > 0$ for $t_1 > 0$ and $t_1 > t_1'$, t_1^* must be a unique positive root. Additionally, t_1^* can easily be estimated as closely as desired. Merely proceed by trial and error while remembering that $t_1^* > 0$ and $t_1^* > t_1'$, and that $\frac{dQ}{dt_1} > 0$ for $t_1 > 0$ and $t_1 > t_1'$. Thus, lower and upper bounds for t_1^* can immediately be established and then revised closer and closer to the true value of t_1^*. The one exception to this procedure would arise when there does not exist even the single positive root of the critical equation. This could happen if and only if

$$\frac{d}{dt_1}(E\{C(t_1)\}) < 0 \qquad \text{for} \quad t_1 > 0 \quad ,$$

which in turn occurs if and only if

$$C_D(N-1)e^{-\mu t_o} - \left[\frac{K}{Ne^{-\mu t_o}} + \frac{C_R}{e^{-\mu t_o}} + C_D(N-1)e^{-\mu t_o}\left(\frac{\theta}{2\mu+\theta}\right) + C_D\,(1-2N)\left(\frac{\theta}{\mu+\theta}\right)\right] < 0.$$

For this case, the obvious recommendation is $t_1 = \infty$, i.e., never take corrective action. The final important implication of the results is that, if the unique positive root t_1^* exists, then since $Q < 0$ if $0 < t_1 < t_1^*$ and $Q > 0$ if $t_1 > t_1^*$, and since $\frac{d}{dt_1}(E\{C(t_1)\})$ has the same sign as Q for $t_1 > 0$, $E\{C(t_1^*)\}$ must be a global minimum.

In summary, if

$$C_D(N-1)e^{-\mu t_o} - \left[\frac{K}{Ne^{-\mu t_o}} + \frac{C_R}{e^{-\mu t_o}} + C_D(N-1)e^{-\mu t_o}\left(\frac{\theta}{2\mu+\theta}\right) + C_D(1-2N)\left(\frac{\theta}{\mu+\theta}\right)\right] < 0 \ ,$$

never take corrective action. Otherwise, select $t_1 = t_1^*$, where t_1^* is the necessarily unique positive root of the critical equation, $Q = 0$. This policy will minimize $E\{C(t_1)\}$. Before solving for t_1^*, determine if

$$\left[2(N-1)\left(1-e^{-\mu t_o}\right) + 1\right] \ C_D \geq C_R\left(\frac{\mu+\theta}{\theta}\right) \ .$$

If so, then $\frac{dQ}{dt_1} > 0$ for $t_1 > 0$. If not, determine t_1', or at least a lower bound to t_1', where

$$2C_D(N-1)\left(1-e^{-\mu(t_o+t_1')}\right) + C_D = C_R\left(\frac{\mu+\theta}{\mu}\right) ,$$

in which case $t_1^* > t_1'$ and $\frac{dQ}{dt_1} > 0$ for $t_1 > t_1'$. In either case, $Q < 0$ for $t_1 = \max\ \{0, t_1'\}$. Then, letting $\max\ \{0, t_1'\}$ be an initial lower bound for t_1^* and using the fact that $\frac{dQ}{dt_1} > 0$ for $t_1 > \max\{0, t_1'\}$, proceed by trial and error or by more systematic numerical methods, to converge upon the true value of t_1^* to whatever accuracy is desired.

6. Example

A certain company in the electronics industry uses an assembly line for assembling one of its products. One of the initial inputs of this assembly line is a certain kind of electronic tube. This tube arrives from an earlier set of operations in lots of 100, usually at a uniform rate of ten lots each week. Unfortunately, due to a number of reasons such as the breakdown of certain equipment, an occasional lot is prevented from being delivered when needed. It has been observed that the failure of the lot to arrive on schedule is a seemingly random event in time, and that the average time between these events is 20 weeks. The consequence of not having any of these tubes available is that the assembly line must shut down. To safeguard against this very serious situation, a spare lot of the tubes is kept in reserve at a centralized storage station. Thus, whenever the lot in the regular flow of production is delayed or rejected, the spare lot is immediately removed from storage and used in place of the regular lot. The new spare lot is then produced during overtime work if necessary.

It has been observed that, ordinarily, the proportion of the tubes which are produced defective is negligible. From previous experience, both within and outside the company, it is estimated that the storage life of this tube is exponentially distributed with a mean of 180 weeks.

Therefore, to safeguard against using the spare lot when it contains an excessive number of defectives, it is felt that the lot should periodically be screened and the defective tubes replaced with non-defective tubes. However, it is not known just how often this corrective action should be taken so as to minimize the total expected cost.

It is estimated that the cost of inspecting the entire lot is \$100.00. The replacements for the defective items would be produced and inspected during overtime work, which would involve a cost of \$10.00 per item. Since there is not enough time to inspect the lot when it is called into service, any defective tube present would be assembled into the corresponding item of the product. The result would be that the assembled product would not test out; it would then be checked, partially disassembled, the tube replaced, and reassembled. The total extra cost involved for each defective tube is approximately \$100.00.

This data is sufficient to permit the use of the results presented earlier to provide a solution. The cost functions clearly are $D(n) = 100n$ and $R(n) = 100 + 10n$. Letting the unit of time be the week, $\mu = 1/180$ and $\theta = 1/20$. Since the proportion of the tubes which are produced defective is negligible, $t_o = 0$. Thus, in the notation of the linear model of Section 4, $A = \frac{800}{711}$ and $(A - \frac{\mu}{\theta}) = \frac{721}{711}$. Referring to Figure 4.1, it is seen that μt_1 should lie between 0.05 and 0.10. Linearly interpolating between the two curves, $\mu t_1 = 0.055$. Therefore, t_1, the elapsed time between consecutive inspections of the same lot in storage, should be about 9.9 weeks.

7. Discussion of Results

All of the foregoing results have been based on the assumption that the particular exponential distribution of storage life is known. Therefore, the following statement may at first seem surprising and contradictory. It is that the widest application of these results probably should be to the situation where the distribution of storage life must be judged to be unknown, at least at the initiation of the program. To understand why, it should be recognized that, for this situation, a program of surveillance sampling would or should be conducted which would have the effect of gradually identifying the underlying distribution of storage life. Thus, it is entirely feasible that, by the time corrective action should be taken, the surveillance sampling can sufficiently identify the underlying distribution of storage life to signal the need for this corrective action with the use of the foregoing results. In this connection, it should be noticed that when the optimum t_1 was being solved for, it was really μt_1 that was being found for a particular ratio $\frac{\mu}{\theta}$. Thus, since $1-e^{-\mu(t_1+t_o)}$ is the expected fraction defective in the lot at time t_1, and since μt_o is chosen so that $1-e^{-\mu t_o}$ is the expected initial fraction defective, the result of the analysis is to determine the value of expected fraction defective at which corrective action should be taken. Hence, when surveillance sampling indicates or predicts approximately this fraction defective while estimating the corresponding ratio $\frac{\mu}{\theta}$, then corrective action would appropriately be taken at the predicted time. In summary, whenever the particular distribution of storage life is unknown, but assumed to be exponential, use surveillance sampling in effect to estimate the particular distribution and then apply the corresponding results.

One limitation of the results is that the distribution of storage life is assumed to be exponential. However, this should at least serve as a useful approximation in a large proportion of the cases. Even when the distribution of storage life is known to be not nearly exponential, the results might be enlightening and instructive. It should also be recognized that, especially when corrective action will consist of replacing the lot, only the early part of the distribution of storage life may really need to resemble the exponential distribution.

Another limitation of the results is that solutions are readily obtainable only for a limited number of $D(n)$ and $R(n)$ functions. Fortunately, among these functions are the extremely important ones studied in the preceding sections. The $R(n)$ function studied appears to be at least a reasonable approximation for most situations of interest. The two $D(n)$ functions studied appear to be reasonable approximations for the two primary types of situations. One of these types is where the imputed cost of defective items in the lot at the time of the emergency is roughly proportional to the number of such defectives. This would occur, for example, where the results of the use of the items in the lot are relatively independent. The other primary type of situation is where the imputed cost of defectives when the emergency occurs increases rapidly as the number of these defectives increases. This would occur, for example, where a few defectives in the lot would be easily compensated for by the many non-defectives but the usefulness of the lot would become seriously compromised by the presence of a relatively large number of defectives. The results derived for the quadratic D(n) function should be useful for this type of situation either as a preliminary result requiring adjustments or as a final solution. The linear D(n) function results should similarly be useful for the first type of situation discussed.

The model does not consider the possibility of the obsolescence of the items in storage or of the termination of the time in which the relevant emergency can occur. However, unless such an occurrence is expected soon, it does not appear that the accuracy of the results would be affected significantly.

8. Conclusions

The results presented here should be helpful in many cases in determining when corrective action should be taken because of the deteriorating quality of a lot in storage. The model does assume the knowledge of considerable information, including the common distribution of storage life of the items in the lot. However, it appears to be feasible to gather and refine this information by surveillance sampling while simultaneously applying the indicated results. Another important limitation is the assumption that the distribution of storage life is exponential. Nevertheless, this assumption should be entirely appropriate in many cases and a useful approximation in many others, at least for instructive preliminary analysis. One of the sets of cost functions analyzed and shown to lead easily to a solution should be appropriate for most cases of interest.

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